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13. ABSTRACT (Maximum 200 words)

This program is developing constructive methods for certain constrained optimization problems arising in the design and control of electromagnetic fields and in the identification of scattering objects. The problems addressed fall into three categories: (i) the design of antennas with optimal radiation characteristics measured in terms of directivity; (ii) the control of the electromagnetic scattering characteristics of an object, in particular the minimization of its radar cross section, by the choice of material properties, and (iii) the determination of the shape of scattering objects with various electromagnetic properties from scattered field data. The main thrust of the program is toward the development of constructive methods based on the use of complete families of solutions of the time-harmonic Maxwell equations in the infinite domain exterior to the radiating or scattering body. During the course of the work an increasing amount of attention has been devoted to the use of iterative methods for the solution of varilous direct and inverse r oblems. The continued investigation and development of these methods and their application in parameter identification has become a significant part of the program.

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Final Report

AFOSR Grant #86–0269

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1. *General Program Description*

The program outlined in our original proposal is directed to the development of constructive methods for certain constrained optimization problems arising in the design and control of electromagnetic fields and in the identification of scattering objects. The problems to be addressed fall into three categories: (i) the design of antennas with optimal radiation characteristics measured in terms of directivity; (ii) the control of the electromagnetic scattering characteristics of an object, in particular the minimization of its radar cross section, by the choice of material properties; and (iii) the determination of the shape of scattering objects with various electromagnetic properties from scattered field data. As originally envisaged, the main thrust of the program is toward the development of constructive methods based on the use of complete families of solutions of the time-harmonic Maxwell equations in the infinite domain exterior to the radiating or scattering body. During the course of the work an increasing amount of attention has been devoted to the use of iterative methods for the solution of various direct and inverse problems. The continued investigation and development of these methods and their application in parameter identification has become a significant part of the program.

2. *Research Activity*

Considerable attention during the course of the program has been directed toward the application of optimization techniques to the problem of determining currents on conformal antennas which optimize various functionals of the radiated

field such as directivity and signal–to–noise ratio. Physical limitations on the design of radiating structures impose certain restrictions. One consequence of which is that the optimization problems are subject to constraints.

Our work (see e.g. [4], [26]) has demonstrated the feasibility of employing complete families of solutions in these constrained optimization problems in which the boundary data serve as control parameters. Indeed, not only have we done a basic analysis of this class of problems, but also have developed computer codes for particular problems.

We were invited to prepare a review of these methods for a special issue of the Proceedings of the IEEE and a manuscript will appear shortly which demonstrates, both theoretically and computationally, how a variety of antenna performance criteria may serve as cost functionals for the general approach. A copy of our review paper is attached. Dissertation work in this area has been carried out by two students, S. Fast and T. Bazow. The Ph.D. thesis of Fast *An Optimization Method for Solving a Radiation Direction Problem*, included codes for the three dimensional electromagnetic problem of directing a signal into a prescribed three dimensional sector by appropriate choice of surface current. T. Bazow, who is currently finishing his work, is studying the more complicated problem of using surface impedance as the control parameter for the problem of controlling the scattered field due to an incoming plane wave. Unlike the acoustic and electromagnetic cases studied earlier, the problem is not reducible to a generalized eigenvalue problem. The constructive procedure for finding optimal impedances is therefore considerably more difficult and is again based on the use of complete families of solutions.

Such optimal radiation problems, if meaningfully formulated, fall naturally into the class of constrained optimization problems. These constraints typically arise in

two ways: inherent constraints due to physical limitations of the system e.g. bounds on input power, and artificial constraints whose introduction is motivated by the desire to control unwanted effects. For example, it has long been recognized that the narrow focusing of the main beam of an antenna has the concomitant (and undesirable) effect of increasing near field power. Indeed one typically wishes to *both* focus the wave beam *and* minimize the power stored in the wave field. Thus a typical problem that arises in antenna design is that of dealing with several possibly conflicting goals.

Such problems may be cast in the framework of multi–criteria optimization. Apparently, such a formulation has not been made for problems in the design of radiating structures although the techniques are known and have been applied to a variety of engineering design problems especially in mechanical engineering. We have prepared a manuscript [41] proposing the use of these techniques and giving examples of numerical calculations for a multi–criteria problem related to that of maximizing the signal–to–noise ratio.

We have also been concerned with establishing existence, uniqueness and continuous dependence results for certain "non–classical" boundary value problems in both the acoustic and electromagnetic problems of time–harmonic scattering. Thus, in the acoustic case we have considered the so–called resistive and conductive "boundary" conditions to model scattering from thin shell–like structures. This work has been extended to electromagnetic problems where our formulation in fact generalized the usual conditions for the transmission problem. In addition to well–posedness we have also considered properties of the set of far field patterns. This work is reported in [7], [13], [14], [29], [34].

Each of these topics of boundary, or more accurately generalized transmission,

conditions is intended to more accurately model the behavior of scattering surfaces which are penetrable to electromagnetic radiation and serve as better approximations to the physical situation of scattering from control bodies. The use of such coatings, modelled by boundary data such as resistivity, for the control of scattered radiation represents an area of mathematical research comparatively new and of potentially significant application. The work done under this grant is preparatory to the study of optimizing scattering characteristics, e.g. the minimization of radar cross section, by choice of impedance, conductivity or resistivity.

We have also devoted considerable effort to inverse problems, both of shape reconstruction and profile inversion. With respect to the former problem, we have developed a constructive method using complete families of radiating solutions. These families are used to formulate the inverse problem of choosing the shape whose scattering characteristics best approximate the measured far field data as a penalized optimization problem whose penalization term involves only the defect in satisfying the boundary conditions. In collaboration with B. Kok, we have developed a working computer code for the problem of reconstructing the shape of a perfectly conducting cylinder when illuminated by an E–polarized incident electromagnetic wave. Work is being completed by X. Jiang on a three–dimensional code for the TM case and a manuscript describing the algorithm and the computational results is being prepared. Results of this aspect of the work have been reported in [4], [6], [20], [21], [25], [26].

The importance of a good initial guess in the shape reconstruction algorithm has become apparent in the course of the numerical work, particularly in the full three dimensional code. One way to obtain a reasonable starting surface is to carry out the inversion in a simpler setting. This can be done at low frequencies

and work was done for reconstruction problems in the low frequency regime, as well as for solutions to the direct problem. Not only has an easily programmable algorithm been developed to find the best ellipsoidal approximation to an unknown scattering shape, but, remarkably, it was found that Kelvin inversion could be used for solutions of the Helmholtz equation, contrary to accepted belief, thus enlarging the class of surfaces for which solutions could be found. These methods can then be used to refine the estimate of the scattering surface used as a starting point in the inverse problem. This work on Kelvin inversion has appeared in two papers by G. Dassios and R. E. Kleinman [5], [9].

The algorithm for reconstruction of the best approximating ellipsoid, originally formulated for the case where measurements of the entire far field are available [1], has been recently modified for the case in which only backscattered data is available, and a computer code has been written and tested by X. Jiang, using a forward code based on the exact low frequency result for ellipsoids. We intend to continue testing with experimental data supplied by colleagues at the University of Michigan Radiation Laboratory who are collaborating under an NSF grant. Work is now in progress to compile this algorithm with the original inverse scattering code to provide a good initial surface for the reconstruction algorithm.

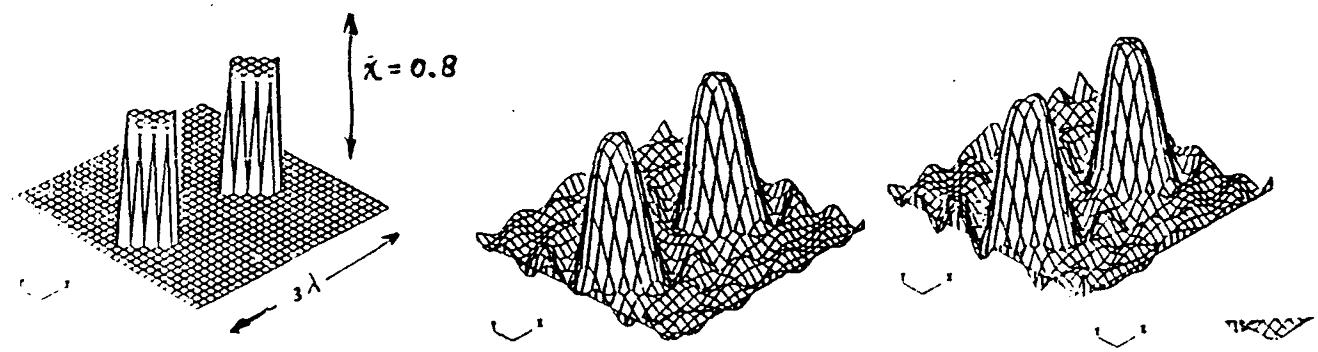
Work continued on the second class of inverse problems involving parameter estimation. In particular we considered the problem of reconstructing an unknown complex index of refraction from known scattered field data. This problem of electromagnetic tomography or profile inversion has received considerable attention in recent years. In its usual formulation it is a constrained nonlinear optimization problem in which both the field and the index of refraction are unknown in the scattering object. Moreover the field depends on the index of refraction in a highly nonlinear

way. Most attacks on this problem embody two principles: first a linearization of the nonlinear dependence and secondly a regularization of the ill-posed optimization problem. The process is an iterative one which requires the direct solution of a scattering problem at each step in order to determine the updated field.

Our approach was motivated by our work on iterative solutions of the integral equations based on error minimization. This work resulted in a uniform framework which encompassed a number of methods – successive over relaxation, conjugate gradient and Krylov methods. Applications of these various methods to the integral equations of scattering was also considered. This work is described in [2], [3], [8], [10], [11], [12], [23], [24], [27], [30], [31], [33], [35], [39].

One of the simplest methods in this class is successive over relaxation where at each step the relaxation parameter is determined so as to minimize residual error. We incorporated this idea in the inverse problem by adding the constraint, the state equation, as a regularizer. We then proposed a modified gradient method to solve the optimization problem. Both the field and the profile were updated at each step by adding a relaxation parameter times an updating function (direction). The direction for the field was taken to be the residual error in the state equation, exactly as in the over-relaxation solution of the direct problem, while the direction for the index of refraction was taken to be the gradient of the error in matching the measured data. The relaxation parameters were found simultaneously by minimizing the regularized functional. This required the solution of nonlinear algebraic equations for the unknown parameters arising from the necessary conditions for a minimum. The novel features of this approach are the elimination of the need to solve a direct problem at each step while retaining the nonlinear nature of the cost functional. This work was carried out in collaboration with P. M. van den Berg of

Delft University of Technology in the Netherlands. Preliminary numerical results are very promising and have been reported in [16] and [37]. Recent results on two dimensional objects indicate that the method is capable of reconstructing discontinuous profiles even with noisy data. The following figures show an original profile and the reconstructions after 64 iterations where the data consisted of the scattered field at 30 points equally spaced on a circle of radius 3λ produced by a line source located successively at each of the 30 locations.



original | reconstruction no noise | reconstruction 10% noise

A manuscript describing these results is under preparation.

4. *Papers and Presentations on AFOSR Grant No. 86-0269*

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2. The Conjugate Gradient Spectral Iterative Technique for Planar Structures, (P. van den Berg and R. E. Kleinman), *IEEE–Trans Antennas and Prop.*, **36**, 1988, 1418–1423.

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4. A Constructive Method for Identification of an Impenetrable Scatterer, (T. S. Angell, R. E. Kleinman, B. Kok and G. F. Roach), *Wave Motion*, **11**, 1989, 185–200.

5. On Kelvin Inversion and Low-Frequency Scattering (G. Dassios and R. E. Kleinman), *SIAM Review*, **31**, 1989, 565–585.

6. Target Reconstruction from Scattered Far Field Data, (T. S. Angell, R. E. Kleinman, B. Kok and G. F. Roach), *Annales des Télécommunications*, **44**, 1989, 456–463.

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8. Iterative Solution of Integral Equations in Scattering Problems, (R. E. Kleinman and P. M. van den Berg), IUTAM Symposium on Elastic Wave Prop. and Non–Destructive Eval., Boulder, CO, July 1989, published in *Elastic Waves and Ultrasonic Nondestructive Evaluation*, S. K. Datta, J. D. Achenbach, Y. S. Rajapakse, eds., Elsevier Science Publ. (North Holland) 1190, 57–62

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12. A Convergent Born Series for Large Refractive Indices, (R. E. Kleinman, G. F. Roach, and P. M. van den Berg), *J. Opt. Soc. Amer. A*, **7** (5), 1990, 890–897.

13. The Helmholtz Equation with Resistive Boundary Conditions, (T. S. Angell, R. E. Kleinman and F. Hettlich), to appear in *SIAM J. Applied Math.*

14. The Conductive Boundary Condition for Maxwell's Equations, (T. S. Angell and A. Kirsch), in preparation.

15. Residual Error – A Simple and Sufficient Estimate of Actual Error in Solutions of Boundary Integral Equations, (G. C. Hsiao, R. E. Kleinman, R. -X. Li, and P. M. van den Berg), in *Computational Engineering with Boundary Elements*, Vol I, S. Grilli, C. A. Brebbia and A. H. -D. Cheng, eds. Computational Mechanics Pub. Southampton, 1990, 173–83.

16. A Non–Linearized Approach to Profile Inversion, (R. E. Kleinman and P. M. van den Berg), *Int. J. of Imaging Systems and Technology* **2**, 1990, 119–126.

17. A Constructive Method for Shape Optimization: A Problem in Hydromechanics, (T. S. Angell and R. E. Kleinman), *IMA J. Appl. Math.*, to appear.

18. Antenna Control and Optimization, (T. S. Angell, R. E. Kleinman and A. Kirsch), in preparation.

19. Single Integral Equations for Scattering by a Penetrable Obstacle, (R. E. Kleinman and P. A. Martin), IUTAM Symposium on Advanced Boundary Element Methods, San Antonio, Texas, April 1987.

20. A Penalty Method for Inverse Problems for Cylindrical Structures, (T. S. Angell and R. E. Kleinman), IEEE-AP/URSI International Symposium, Blacksburg, VA, June 1987.

21. Some Recent Methods for Three Dimensional Inverse Scattering, (T. S. Angell and R. E. Kleinman), XXII General Assembly, URSI, Tel Aviv, Israel, August 1987.

22. Some New Boundary Integral Equations in Scattering by Penetrable Obstacles, (R. E. Kleinman), NSF-CBMS Regional Conference, Mathematical Foundation of the Boundary Element Method, University of Kentucky, May 1988.

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24. An Iterative Solution to Acoustic Scattering by Rigid Bodies, (R. E. Kleinman and J. Shirron), 17th International Congress on Theoretical and Applied Mechanics, Grenoble, France, August 1988.

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27. An Over-Relaxation Method for the Iterative Solution of Integral Equations in Scattering Problems, (R. E. Kleinman, G. F. Roach, L. S. Schuetz, J. Shirron, and P. M. van den Berg), National Radio Science Meeting (URSI), Boulder, CO, January 1989.

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32. Integral Equations for Electromagnetic Scattering from an Indented Screen, (J. S. Asvestas and R. E. Kleinman), National Radio Science Meeting (URSI), Boulder, CO, January, 1990.

33. Iterative Methods for Solving Integral Equations in Continuum Mechanics, (R. E. Kleinman), DFG Workshop "Engineering and Mathematical Analysis of Fracture and Inelastic Problems," Lambrecht, Federal Republic of Germany, January, 1990.

34. Resistive and Conductive problems in Classical Scattering, (T. S. Angell), Oberwolfach Conference Methoden und Verfahren der Mathematischen Physik, December, 1989.

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36. Some Questions in Optimal Design of Floating Bodies, (T. S. Angell and R. E. Kleinman), Fifth International Workshop on Water Waves and Floating Bodies, Manchester, England, March, 1990.

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38. Integral Equation Techniques, (C. M. Butler, R. E. Kleinman, D. R. Wilton), XXIII General Assembly of URSI, Prague, Czechoslovakia, August-September 1990.

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40. A Hybrid Method for Two-Dimensional Problems in Tomography, (R. E. Kleinman and P. M. van den Berg), *J. Comp. and Appl. Math.*, to appear.

41. Multicriteria Optimization in Antenna Design, (T. S. Angell and A. Kirsch), submitted for publication.

42. Two Dimensional Profile Inversion, (R. E. Kleinman and P. M. van den Berg), International Conference on Inverse Problems - Computational Algorithms, Texas A & M Univ., March, 1991.